Ejemplos:

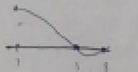
① Pol. Interp.

t	1	3	8
f	10	20	54

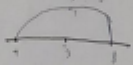
$p(t) = f_1 L_1(t) + f_2 L_2(t) + f_3 L_3(t) = \frac{5(t-3)(t-8)}{14} + \frac{5(t-1)(t-8)}{3} + \frac{54(t-1)(t-3)}{27}$

$p(2) = \frac{90}{21} + 70 - \frac{750}{21} = \frac{25}{7}$

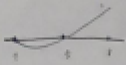
$L_1(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)} = \frac{(t-3)(t-8)}{28}$



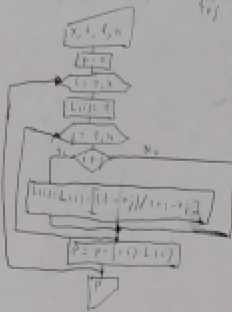
$L_2(t) = \frac{(t-1)(t-8)}{(2)(-5)} = -\frac{(t-1)(t-8)}{10}$



$L_3(t) = \frac{(t-1)(t-3)}{27}$



② $p = \sum_{i=1}^{n} f_i \cdot L_i = \sum_{i=1}^{n} \left(f_i \cdot \prod_{j \neq i} \frac{t - x_j}{x_i - x_j} \right)$



③ Alm. puntos

Incógnitas: $\begin{pmatrix} a_1 \\ a_2 \\ a_n \end{pmatrix}$

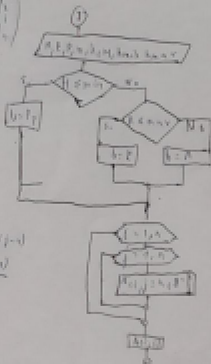
$p(x_i) = p_i$

$p(x) = a_1 + a_2 x + a_3 x^2 \dots a_n x^{n-1}$

$p_i \begin{cases} h_i \\ t_i, f_i \end{cases}$ $q(h_i) = f_i$

$\begin{cases} a_1 + a_2 h_1 + a_3 h_1^2 \dots a_n h_1^{n-1} = f_1 \\ a_1 + a_2 h_2 + a_3 h_2^2 \dots a_n h_2^{n-1} = f_2 \\ a_1 + a_2 h_n + a_3 h_n^2 \dots a_n h_n^{n-1} = f_n \end{cases}$

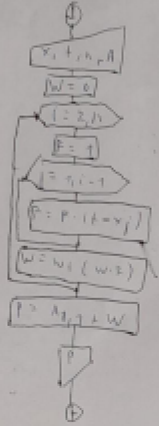
$\begin{pmatrix} 1 & h_1 & h_1^2 & h_1^{n-1} \\ 1 & h_2 & h_2^2 & h_2^{n-1} \\ 1 & h_n & h_n^2 & h_n^{n-1} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}$ $A(i,j) = h_i^{j-1}$



④ $A(1,1) = W$

$p = \sum_{i=2}^{n} A(1,i) \cdot F$

$p = \sum_{i=1}^{n} f[x_1 \dots x_i] \cdot \prod_{j=1}^{i-1} (t - x_j)$



⑥ Dif. divididas:

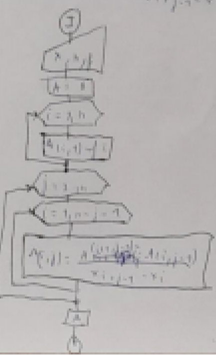
$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 \\ A_{41} & A_{42} & 0 & 0 & 0 \\ A_{51} & 0 & 0 & 0 & 0 \end{pmatrix}$

$j = 2, \dots, n$

$i = 1, \dots, n-(j-1)$

$n - j - 1$

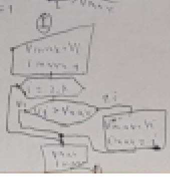
$n - (j-1)$ $f_i = A(i,j) = \frac{A(i+1, j-1) - A(i, j-1)}{x_{i+j-1} - x_i}$



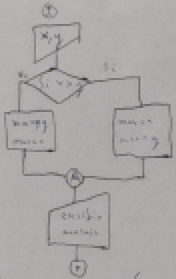
⑤ $i = 1, \dots, n$ $\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$

vmax = v_1 ... v_2 > vmax

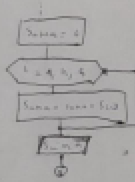
imax = 1



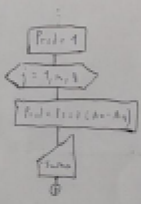
Condiciones.



Sumatoria



Productorio

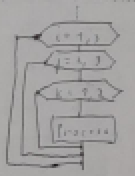


- Cumplir regla de asignación: → valor izquierda, → y lo guarda.
- Hacer tabla de seguimiento.

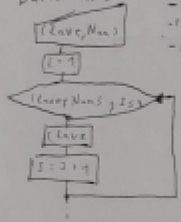
Reglas:

- Sentencia igual: sin valor en derecha a no hay entrada de datos (variables) definidas en un algoritmo antes.
- Orden:
  - copiar fórmula
  - crear bucle por cada subíndice (variable) a la izquierda.
  - Ver datos a la derecha para dar info al subíndice.
  - Dar esa info en la izquierda.

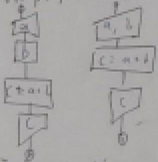
Bucles anidados (Ciclos triples)



Bucle While



Simplificación



Algunos ejercicios:

$$P = \sum_{i=1}^{n} \left\{ \sum_{j=2}^{n} \left[\prod_{k=1}^{p} \left(\frac{i+k+2k}{j-1} \right) \right] \right\}$$

$$V[i] = \sum_{k=1}^{n} a[k] \Big/ \prod_{k=1}^{n} r[k] + i$$

Interpolación:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$y_i = \frac{b_i - \sum_{k=i+1}^{n} a_{ik} x_k}{a_{ii}}$$

Si $a_{n\times n}$ (matriz triang. sup)

$a(1,1) \cdot y_1 = b_1 \rightarrow y_1 = \frac{b_1}{a(1,1)}$

$$y_{pos} = \frac{(b_i - a_{21} \cdot y_1)}{a_{22}}$$

$$y_{pos} = \frac{b_2 - (a_{31} \cdot y_1 + a_{32} \cdot y_2)}{a_{33}}$$

$$y[i] = \frac{b_i - \sum a_{ik} (a_{12} \cdot y_2 + a_{13} \cdot y_3 + a_{14} \cdot y_4)}{a_{11}}$$

$$f(x_1) = \sum_{i=1}^{n-1} f[x_1, \ldots, x_i] \prod_{j=1}^{i-1}$$

Interpolación polinómica de Lagrange.

$P_{(x)} = a_1 + a_2 x + a_3 x^2 + \ldots a_n x^{n-1}$ si $\{x_i\}_{i=1}^{n}$

Fórmula de Newton: $p(x) = f[x_1] + f[x_1, x_2](x - x_1) + f[x_1, x_2, x_3] \ldots + f[x_1, x_2, \ldots, x_n](x - x_1) \ldots (x - x_{n-1})$ ⊛

Según Lagrange: $p(x_i) = f_i \quad i(1, \ldots, n)$

Método de diferencias divididas:

- Orden 0: $f[x_i] = f(x_i) = f_i$
- Orden 1: $f[x_i, x_j] = \frac{f_j - f_i}{x_j - x_i}$
- Orden 2: $f[x_i, x_j, x_k] = \frac{f[x_j, x_k] - f[x_i, x_j]}{x_k - x_i}$
- Orden i: $f[x_1, x_2, \ldots, x_{i+1}] = \frac{f[x_2, \ldots, x_{i+1}] - f[x_1, \ldots, x_i]}{x_{i+1} - x_1}$

Polinomios base de Lagrange.

$$L_{i(x)} = \prod_{j=1}^{n} \frac{x - j}{x_i - x_j}$$

$p(x_1) = f_1 L_1(x_1) + f_2 L_2(x_1) + \ldots + f_n L_n(x_1) = f_1$

$p(x_2) = f_1 L_1(x_2) + f_2 L_2(x_2) + \ldots + f_n L_n(x_2) = f_2$

$p(x_n) = f_1 L_1(x_n) + f_2 L_2(x_n) + \ldots + f_n L_n(x_n) = f_n$

Ejemplo:

Soporte $\{-1, 0, 3, 5\}$

Función $\{10, 12, 8, 6\}$

$x_1 = -1$, $x_2 = 0$, $x_3 = 3$, $x_4 = 5$

10	$\frac{12-10}{0+1} = 2$	$\frac{-\frac{4}{3} - 2}{3+1} = -\frac{10}{12}$	$\frac{-\frac{10}{12} - \frac{5}{6}}{6} = \frac{-\frac{1}{15} + \frac{5}{6}}{6} = \frac{3}{20}$
12	$\frac{8-12}{3-0} = -\frac{4}{3}$	$\frac{-1+\frac{4}{3}}{5-0} = \frac{1}{15}$	
8	$\frac{6-8}{5-3} = -1$		
6			

$$p(x) = ~~f[x_1] + f[x_2]~~ \; 10 + 2(x-(-1)) - \frac{5(x+1)(x-0)}{6} + \frac{3(x+1)x(x-3)}{20}$$