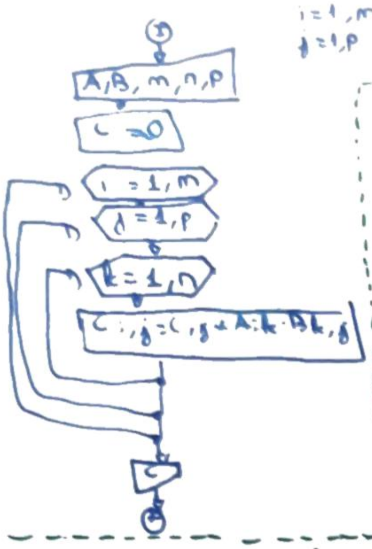
Algoritmos

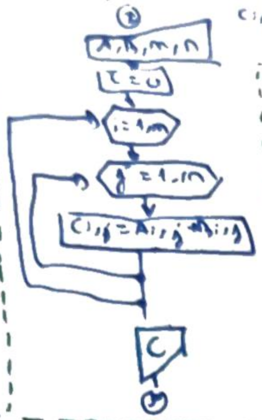
$A_{m,n}$ $\quad B_{n,p}$

$A \cdot B = C \rightarrow C_{m,p}$

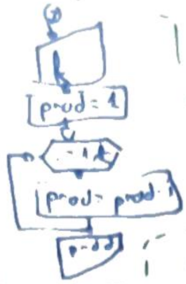
$i=1,m$ $\quad j=1,p$ $\qquad C_{i,j} = \sum_{k=1}^{n} A_{i,k} \cdot B_{k,j}$



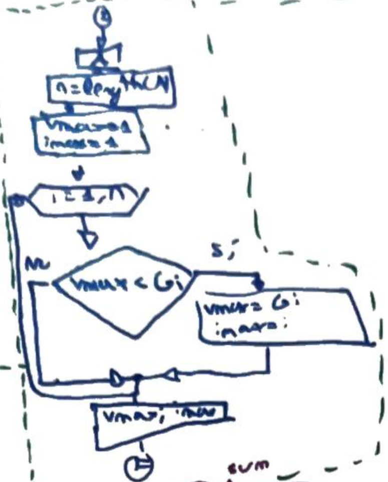
$A_{m,n} + B_{m,n} = C_{m,n}$ $\qquad C_{i,j} = A_{i,j} + B_{i,j}$



$\prod_{k=1}^{k}$



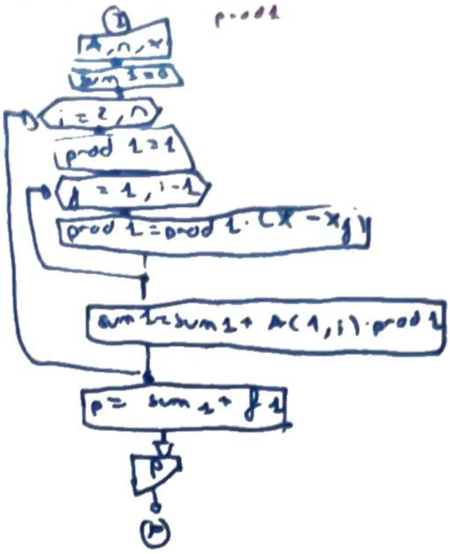
Prod escalar $\rightarrow \sum_{i=1}^{n} W_i \cdot V_i$



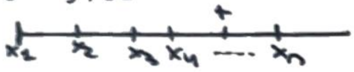
Algoritmo polinomio dif div (norma es el de la tabla con valor que hay que interpolar)

$p = f_1 + \sum_{i=2}^{n} \left(f[x_1, x_2, \ldots x_i] \cdot \prod_{j=1}^{i-1} (x - x_j) \right)$

$A(1,i)$ $\quad$ prod 1



Alg obtener valor interpolado en t con soporte $\{x_i\}_{i=1}^{n}$ para valores $\{f_i\}_{i=1}^{n}$



Algoritmo tabla dif div

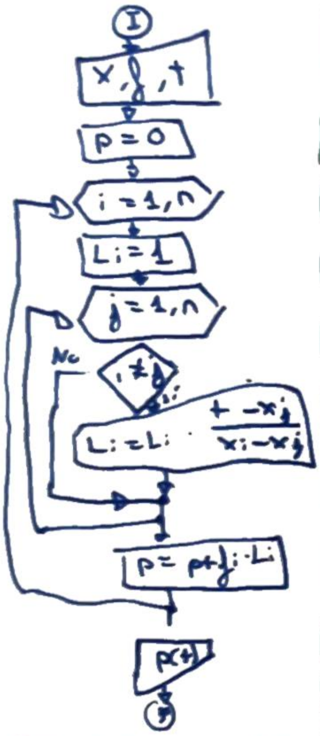
$A(i,1) = f_i$

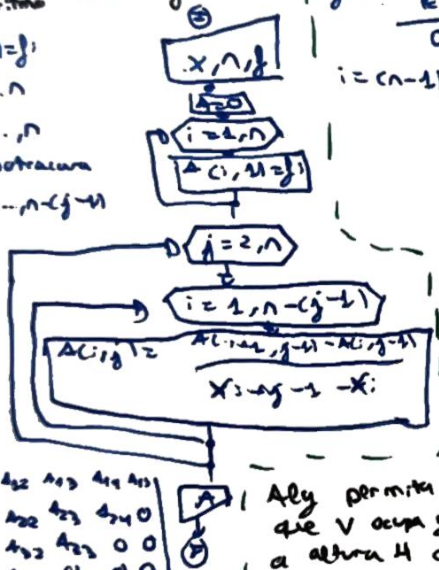
$i = 1 \ldots n$

$j = 2, \ldots, n$

$A(i,j)$ sotracara

$i = 1, \ldots, n-(j-1)$



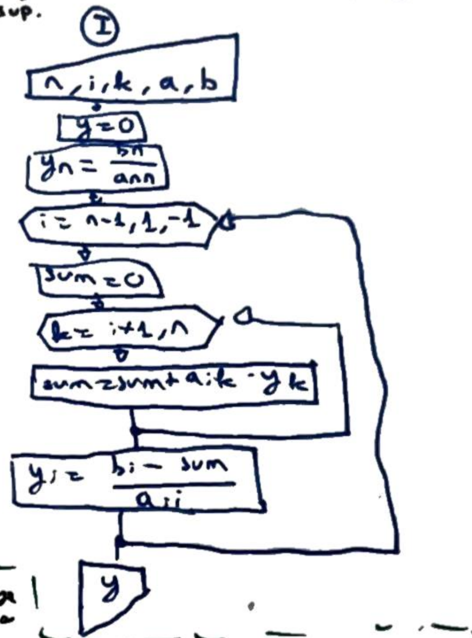


$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 \\ A_{41} & A_{42} & 0 & 0 & 0 \\ A_{51} & 0 & 0 & 0 & 0 \end{pmatrix} = A$$

$y_i = \dfrac{b_i - \sum_{k=i+1}^{n} a_{ik} \, y_k}{a_{ii}}$ (sum)

$i = (n-1), 1, -1$

Alg método remente matriz Δ sup.

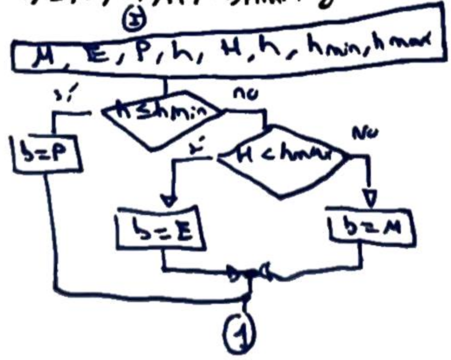


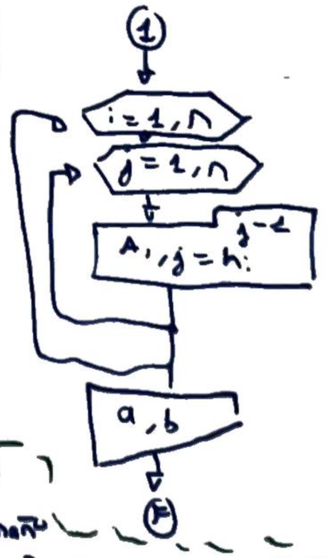
Alg permita apreciar por Lagrange que V ocupa gas de globo aerostático a altura H con 3 opciones de gas: M → V demetano, E → V de etano, P → propano

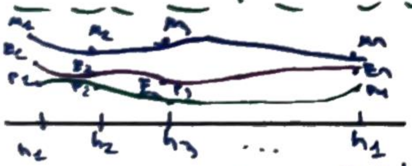
Alturas almacenadas en h.

$h_{max} \leq H$ $\quad H_{min} < H < h_{max}$ $\quad H \leq h_{min}$

Dados M, E, P, h, n, H, hmin y hmax





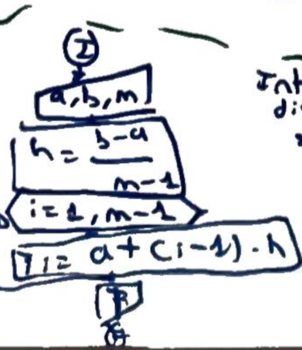


$$\begin{pmatrix} 1 & h_1 & h_1^2 & \ldots & h_1^{n-1} \\ 1 & h_2 & h_2^2 & \ldots & h_2^{n-1} \\ \ldots \\ 1 & h_n & h_n^2 & \ldots & h_n^{n-1} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}$$

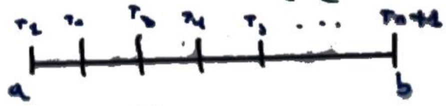
$A \qquad x \qquad b$

$$\begin{cases} a_1 + a_2 h_1 + a_3 h_1^2 + \ldots + a_n h_1^{n-1} = f_1 \\ a_1 + a_2 h_2 + a_3 h_2^2 + \ldots + a_n h_2^{n-1} = f_2 \\ \ldots \\ a_1 + a_2 h_n + a_3 h_n^2 + \ldots + a_n h_n^{n-1} = f_n \end{cases}$$

$A_{i,j} = h_i^{j-1}$



Intervalo $[a,b]$ dividido en m subintervalos iguales de tamaño h



Interpolación — Pablo de la Hoz Andrés

Diferencias divididas

$$f[x_1,x_2] = \frac{f_2 - f_1}{x_2 - x_1} \qquad f[x_1,x_2,x_3] = \frac{f[x_2,x_3] - f[x_1,x_2]}{x_3 - x_1}$$

(2º, último, primero; final → primero)

Grado i: $$f[x_1,x_2,\dots,x_{i+1}] = \frac{f[x_2,\dots,x_{i+1}] - f[x_1,\dots,x_i]}{x_{i+1} - x_1}$$

– Fórmula de Newton → $$f(x_1) + \sum_{i=2}^{n}\left(f[x_1,x_2,\dots x_i] \cdot \prod_{j=1}^{i-1} (x - x_j) \right)$$

– Tabla dif div

x_1	f_1	$f[x_1,x_2] = \frac{f_2-f_1}{x_2-x_1}$	$f[x_1,x_2,x_3] = \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$	$f[x_1,x_2,x_3,x_4] = \frac{f[x_2,x_3,x_4]-f[x_1,x_2,x_3]}{x_4-x_1}$
x_2	f_2	$f[x_2,x_3] = \frac{f_3-f_2}{x_3-x_2}$	$f[x_2,x_3,x_4] = \frac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2}$	
x_3	f_3	$f[x_3,x_4] = \frac{f_4-f_3}{x_4-x_3}$		
x_4	f_4			
soporte	orden 0	orden 1	2	3

$$A(i,j) = \frac{A(i+1, j-1) - A(i, j-1)}{x_{i+j-1} - x_i}$$ → si ponemos la tabla como una matriz

Polinomios de base de Lagrange

$$L_i(x) = \prod_{\substack{j=1 \\ i \neq j}}^{n} \frac{x - x_j}{x_i - x_j}$$

siendo x el valor interpolado (conocido).
$\{x_j\}$ → puntos soporte.
f_i → conocido, y en los soporte.

$$p(x) = \sum_{i=1}^{n} f_i \cdot L_i(x)$$

Conocemos T^a en función de t de un fluido. Obtener por interp Lagrange valor interp en $t=2$ y representar gráficamente los polinomios de base.

t	1	5	8
T	10	20	50

$$p(t) = T_1 \cdot L_1(t) + T_2 \cdot L_2(t) + T_3 \cdot L_3(t)$$

$$L_1(t) = \frac{(t-t_2)(t-t_3)}{(t_1-t_2)(t_1-t_3)} = \frac{(t-5)(t-8)}{28} \qquad L_2(t) = \frac{(t-t_1)(t-t_3)}{(t_2-t_1)(t_2-t_3)} = -\frac{(t-1)(t-8)}{12}$$

$$L_3(t) = \frac{(t-t_1)(t-t_2)}{(t_3-t_1)(t_3-t_2)} = \frac{(t-1)(t-5)}{21}$$

$$p(t) = \frac{5}{14}(t-5)(t-8) - \frac{5}{3}(t-1)(t-8) + \frac{50}{21}(t-1)(t-5)$$

$p(2)$ = sustituir

Ej dif div — soporte $\{-1,0,3,5\}$ — Función $\{10,12,8,6\}$

-1	10	$\frac{12-10}{0-(-1)} = 2$	$\frac{-\frac{4}{3}-2}{3+1} = -\frac{5}{6}$	$\frac{\frac{1}{15}-(-\frac{5}{6})}{5-(-1)} = \boxed{\frac{3}{20}}$
0	12	$\frac{8-12}{3-0} = -\frac{4}{3}$	$\frac{-1-(-\frac{4}{3})}{5-0} = \frac{1}{15}$	
3	8	$\frac{6-8}{5-3} = -1$		
5	6			

$$p(x) = 10 + 2\cdot(x-(-1)) - \frac{5}{6}(x+1)(x-0) + \frac{3}{20}(x+1)(x)(x-3)$$

Método de remonte

Δ superior → $$y_i = \frac{b_i - \sum_{k=i+1}^{n} a_{ik} \cdot y_k}{a_{ii}}$$

si $n=4$ $$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$i = (n-1), 1, -1$ → ya que para y_2 hace falta y_3 y así. hay que pr… $a_{44} \cdot y_4 = b_4$ → $y_4 = \frac{b_4}{a_{44}}$

$$y_n = \frac{b_n}{a_{nn}} \qquad y_3 = \frac{b_3 - a_{34} \cdot y_4}{a_{33}}$$

Δ inf → $$y_i = \frac{b_i - \sum_{k=1}^{i-1} a_{ik} \cdot y_k}{a_{ii}} \qquad i = 2, n, 1$$

$$y_2 = \frac{b_2 - (a_{23} \cdot y_3 + a_{24} \cdot y_4)}{a_{22}}$$

$$y_1 = \frac{b_1 - (a_{12} \cdot y_2 + a_{13} \cdot y_3 + a_{14} \cdot y_4)}{a_{11}}$$

Se conoce presión de un gas en varios puntos $\{x_i\}$ estimar presión en punto x en los siguientes casos, sabemos $\{f_i\}$. a) $x_1 = 1$ $x_2 = 3$ $P_1 = 100$ $P_2 = 200$ $x = 2$

a) $$\begin{cases} a_1 + a_2 \cdot x_1 = f_1 \\ a_1 + a_2 \cdot x_2 = f_2 \end{cases} \to \begin{cases} a_1 + a_2 = 100 \\ a_1 + 3a_2 = 200 \end{cases} \to a_1 = 50 \quad a_2 = 50$$

$P(x) = 50(1+x)$ $\quad P(2) = 50 \cdot (3) = \boxed{150}$

b) $x_1 = -1$ $x_2 = 0$ $x_3 = 1$ $P_1 = 80$ $P_2 = 150$ $P_3 = 90$ $x = 2$

$$\begin{cases} b_1 + b_2 \cdot x_1 + b_3 \cdot x_1^2 = 80 \\ b_1 + b_2 \cdot x_2 + b_3 \cdot x_2^2 = 150 \\ b_1 + b_2 \cdot x_3 + b_3 \cdot x_3^2 = 90 \end{cases} \to \begin{cases} b_1 - b_2 + b_3 = 80 \\ b_1 = 150 \\ b_1 + b_2 + b_3 = 90 \end{cases} \to \begin{matrix} b_1 = 150 \\ b_2 = 5 \\ b_3 = -65 \end{matrix}$$

$$P(x) = 150 + 5 \cdot x - 65 x^2$$
$$P(2) = 150 + 10 - 65 \cdot 4 = -100$$