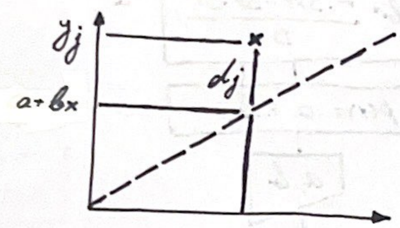
AJUSTE POR
MÍNIMOS CUADRADOS

A partir de un conjunto de valores (x,y) buscamos una recta $p(x)$ que minimice la suma de los cuadrados de las distancias entre la recta y los puntos.



Si la ecuación de la recta es $y=a+bx$, entonces $d_j = y_j - (a+bx)$, que es la distancia desde es punto a la recta (la cual es la que queremos minimizar su cuadrado).
Si tenemos n puntos:

$$\sum_{j=1}^{n} d_j^2 = \sum_{j=1}^{n} (y_j - (a + bS_j))^2 \rightarrow \text{Para minimizarla:}$$

$$\frac{\partial f}{\partial a} = 0 \quad \Big| \quad 2\sum_{j=1}^{n} [(y_j - (a + bS_j))(-1)] = 0 \rightarrow \overbrace{\sum_{j=1}^{n} y_j}^{SY} = \overbrace{\sum_{j=1}^{n} a}^{n\cdot a} + \overbrace{b\sum_{j=1}^{n} S_j}^{bSX}$$

$$\frac{\partial f}{\partial b} = 0 \quad \Big| \quad 2\sum_{j=1}^{n} [(y_j - (a + bS_j))(-S_j)] = 0 \rightarrow \overbrace{\sum_{j=1}^{n} (y_j \cdot S_j)}^{SXY} = \overbrace{\sum_{j=1}^{n} aS_j}^{aSX} + \overbrace{\sum_{j=1}^{n} bS_j^2}^{bSX2}$$

Se pueden extraer las constantes a y b como factor común.

★SIMPLIFICAMOS:

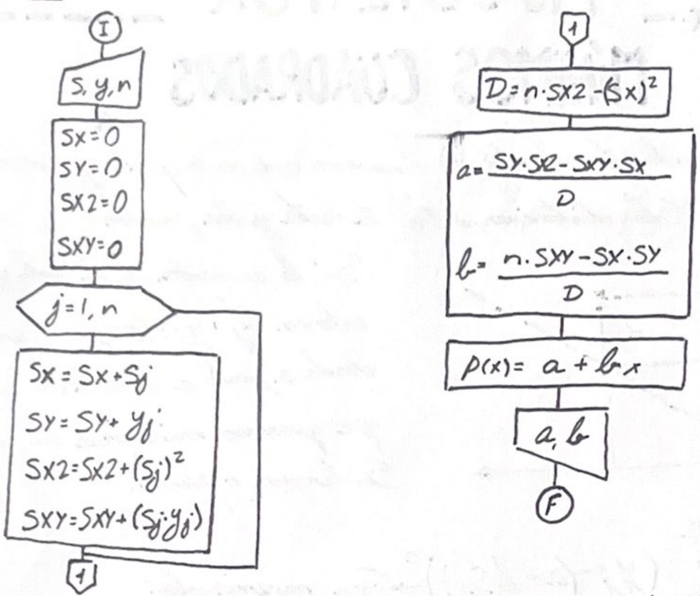
$$\begin{cases} n\cdot a + b\,SX = SY \\ a\cdot SX + b\,SX2 = SXY \end{cases} \xrightarrow{\text{CRAMER}} D = \begin{vmatrix} n & SX \\ SX & SX2 \end{vmatrix} = n\cdot SX2 - (SX)^2$$

$$a = \frac{\begin{vmatrix} SY & SX \\ SXY & SX2 \end{vmatrix}}{D} = \frac{SY\cdot SX2 - SXY\cdot SX}{n\cdot SX2 - (SX)^2} \qquad b = \frac{\begin{vmatrix} n & SY \\ SX & SXY \end{vmatrix}}{D} = \frac{n\,SXY - SX\cdot SY}{n\cdot SX2 - (SX)^2}$$

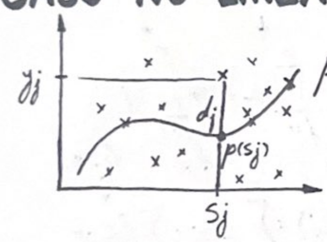
Como $p(x) = a + bx$

$$\boxed{p(x) = \frac{\left(\sum_{j=1}^{n} y_j\right)\left(\sum_{j=1}^{n} S_j^2\right) - \left(\sum_{j=1}^{n} S_j y_j\right)\left(\sum_{j=1}^{n} S_j\right)}{n\cdot\left(\sum_{j=1}^{n} S_j^2\right) - \left(\sum_{j=1}^{n} S_j\right)^2} + \frac{n\left(\sum_{j=1}^{n} (S_j y_j)\right) - \left(\sum_{j=1}^{n} S_j\right)\left(\sum_{j=1}^{n} y_j\right)}{n\cdot\left(\sum_{j=1}^{n} S_j^2\right) - \left(\sum_{j=1}^{n} S_j\right)^2} X}$$

ALGORITMO:



CASO NO LINEAL: $m = n^{\circ}$ de parámetros



$n = n^{\circ}$ de puntos

$$p(x) = a_1 + a_2 x + a_3 x^2 + \dots + a_m x^{m-1} = \sum_{j=1}^{n} d_j^{\,2} = \sum_{j=1}^{n} \left[(y_j - p(s_j))^2 \right]$$

Polinomio de grado $m-1$

Hasta aquí es igual que antes

$$\sum_{j=1}^{n} \left[(y_j - p(s_j))^2 \right] = \sum_{j=1}^{n} \left[\left(y_j - \sum_{k=0}^{m} a_k \cdot (s_j)^{k-1} \right)^2 \right] = f(a_1, a_2, a_3 \dots a_m)$$

Minimizamos m como antes:

$$(1)\quad \frac{\partial f}{\partial a_1} = 2 \sum_{j=1}^{n} \left[\left(y_j - (a_1 + a_2 s_j + a_3 (s_j)^2 \dots + a_m (s_j)^{m-1}) \right) \cdot (-1) \right] = 0 \rightarrow \sum_{j=1}^{n} a_1 + a_2 \sum_{j=1}^{n} s_j + a_3 \sum_{j=1}^{n} s_j^{\,3} + \dots + a_m \sum_{j=1}^{n} s_j^{\,m-1} = \sum_{j=1}^{n} y_j$$

$$(2)\quad \frac{\partial f}{\partial a_2} = 2 \sum_{j=1}^{n} \left[\left(y_j - (a_1 + a_2 s_j + \dots + a_m (s_j)^{m-1}) \right) \cdot (-s_j) \right] = 0 \rightarrow a_1 \sum_{j=1}^{n} s_j + a_2 \sum_{j=1}^{n} s_j^{\,2} + a_3 \sum_{j=1}^{n} s_j^{\,3} + \dots + a_m \sum_{j=1}^{n} s_j^{\,m} = \sum_{j=1}^{n} s_j y_j$$

...

$$(m)\quad \frac{\partial f}{\partial a_m} = 2 \sum_{j=1}^{n} \left[\left(y_j - \sum_{k=0}^{m} a_k \cdot (s_j)^{k-1} \right) \cdot \left(-(s_j^{\,m-1}) \right) \right] = 0 \rightarrow a_1 \sum_{j=1}^{n} s_j^{\,m-1} + a_2 \sum_{j=1}^{n} s_j^{\,m} + a_3 \sum_{j=1}^{n} s_j^{\,m+1} + \dots + a_m \sum_{j=1}^{n} s_j^{\,2m-2} = \sum_{j=1}^{n} s_j^{\,m-1} \cdot y_j$$

Se obtiene un sistema de m ecuaciones y m incógnitas.

$$s_j^{\,m-1} \cdot s_j^{\,m-1} = s_j^{\,2m-2}$$

El sistema de ecuaciones es:

$$\underbrace{S}_{\substack{\text{matriz}\\\text{coeficiente}\\\text{del}\\\text{sistema}}}\cdot\underbrace{A}_{\substack{\text{vector}\\\text{incógnitas}}}=\underbrace{Y}_{\substack{\text{término}\\\text{independiente}}} \Rightarrow \begin{pmatrix} n & \sum_{j=1}^{n} s_j & \cdots & \sum_{j=1}^{n} s_j^{m-1} \\ \sum_{j=1}^{n} s_j & \sum_{j=1}^{n} s_j^{2} & \cdots & \sum_{j=1}^{n} s_j^{m} \\ \sum_{j=1}^{n} s_j^{2} & \sum_{j=1}^{n} s_j^{3} & \cdots & \sum_{j=1}^{n} s_j^{m+1} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{j=1}^{n} s_j^{m-1} & \sum s_j^{m} & \cdots & \sum_{j=1}^{n} s_j^{2m-2} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \cdots \\ a_m \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} y_j \\ \sum_{j=1}^{n} y_j s_j \\ \sum_{j=1}^{n} y_j s_j^{2} \\ \cdots \\ \sum_{j=1}^{n} y_j s_j^{m-1} \end{pmatrix}$$

ALGORITMO: